**Online Supplement**

**Residual estimates from Scenarios 1, 2, and 3**

Intersections written as the sequence of values of through .

|  |  |  |  |
| --- | --- | --- | --- |
| Intersection | Stratum-level Residuals | | |
|  | Scenario 1 | Scenario 2 | Scenario 3 |
| 00000 | 0.246 | 0.241 | -0.507 |
| 00001 | 0.253 | 0.262 | -0.478 |
| 00010 | 0.229 | 0.229 | -0.513 |
| 00011 | 0.247 | 0.212 | -0.468 |
| 00100 | 0.251 | 0.254 | -0.500 |
| 00101 | 0.214 | 0.256 | -0.521 |
| 00110 | 0.241 | 0.260 | -0.503 |
| 00111 | 0.273 | 0.252 | -0.466 |
| 01000 | -0.248 | -0.225 | 0.493 |
| 01001 | -0.219 | -0.274 | 0.505 |
| 01010 | -0.221 | -0.266 | 0.522 |
| 01011 | -0.240 | -0.241 | 0.481 |
| 01100 | -0.247 | -0.207 | 0.458 |
| 01101 | -0.259 | -0.266 | 0.515 |
| 01110 | -0.268 | -0.245 | 0.487 |
| 01111 | -0.233 | -0.230 | 0.516 |
| 10000 | -0.214 | -0.244 | 0.539 |
| 10001 | -0.247 | -0.228 | 0.452 |
| 10010 | -0.280 | -0.211 | 0.469 |
| 10011 | -0.231 | -0.254 | 0.496 |
| 10100 | -0.208 | -0.270 | 0.518 |
| 10101 | -0.289 | -0.258 | 0.493 |
| 10110 | -0.250 | -0.257 | 0.494 |
| 10111 | -0.238 | -0.245 | 0.514 |
| 11000 | 0.247 | 0.251 | -0.543 |
| 11001 | 0.223 | 0.264 | -0.448 |
| 11010 | 0.232 | 0.218 | -0.474 |
| 11011 | 0.226 | 0.265 | -0.510 |
| 11100 | 0.236 | 0.243 | -0.448 |
| 11101 | 0.264 | 0.225 | -0.516 |
| 11110 | 0.260 | 0.226 | -0.498 |
| 11111 | 0.246 | 0.263 | -0.510 |

**R code for Scenarios 1, 2, and 3 in text**

*#For MLM models, follow parameters set by Evans & Erickson (SSM 2019)and Evans (SSM 2019)*  
*#burn-in: 5000 iterations; total iterations:50,000 iterations; thinning: 50 iterations*   
*#sample size=100000*

**library**(rstan)  
**library**(brms)  
  
n = 100000  
seed=12345  
  
############################## Scenario 1 ###################################  
  
**set.seed** (seed)  
  
 Dat<-**data.frame**( x1 = **sample**(**c**(0,1),n, replace = TRUE),  
 x2 = **sample**(**c**(0,1),n, replace = TRUE),  
 x3 = **sample**(**c**(0,1),n, replace = TRUE),  
 x4 = **sample**(**c**(0,1),n, replace = TRUE),  
 x5 = **sample**(**c**(0,1),n, replace = TRUE))  
 Dat**$**cluster <- **factor**(10000**\***(Dat**$**x1**+**1) **+** 1000**\***(Dat**$**x2**+**1) **+** 100**\***(Dat**$**x3**+**1) **+** 10**\***(Dat**$**x4**+**1) **+** 1**\***(Dat**$**x5**+**1))  
  
 Dat**$**int <- Dat**$**x1**\***Dat**$**x2  
 Dat**$**y = Dat**$**x1 **+** Dat**$**x2 **+** Dat**$**x3 **+** Dat**$**x4 **+** Dat**$**x5 **+** Dat**$**int **+** **rnorm**(n)  
   
 ###############OLS REGRESSION without interaction term  
 OLScoeff <- **summary**(**lm**(y **~** x1 **+** x2 **+** x3 **+** x4 **+** x5, data = Dat))**$**coefficients[,1]  
 OLScoeffSE <- **summary**(**lm**(y **~** x1 **+** x2 **+** x3 **+** x4 **+** x5, data = Dat))**$**coefficients[,2]  
 OLScoeff2 <- **cbind**(OLScoeff, OLScoeffSE)  
 OLScoeff2  
   
 ###############OLS REGRESSION with interaction term  
 OLSintcoeff <- **summary**(**lm**(y **~** x1 **+** x2 **+** x3 **+** x4 **+** x5 **+** x1**\***x2, data = Dat))**$**coefficients[,1]  
 OLSintcoeffSE <- **summary**(**lm**(y **~** x1 **+** x2 **+** x3 **+** x4 **+** x5 **+** x1**\***x2, data = Dat))**$**coefficients[,2]  
 OLSintcoeff2 <- **cbind**(OLSintcoeff, OLSintcoeffSE)  
 OLSintcoeff2  
   
 ###############MAIHDA, without interaction term  
 MLM2<- **brm**(formula= y **~** x1 **+** x2 **+**x3 **+** x4 **+** x5 **+** (1**|**cluster), data=Dat, seed=seed, cores=4, iter=50000, warmup = 5000, thin = 50)  
 *#extract main and randoms effects coefficients*  
 **fixef**(MLM2)  
 **ranef**(MLM2)**$**cluster  
   
 ###############MAIHDA, with interaction term  
 MLMint2<- **brm**(formula= y **~** x1 **+** x2 **+**x3 **+** x4 **+** x5 **+** x1**:**x2 **+** (1**|**cluster), data=Dat, seed=seed, cores=4, iter=50000, warmup = 5000, thin = 50)  
 *#extract main effects coefficients*  
 **fixef**(MLMint2)  
 **ranef**(MLMint2)**$**cluster  
  
  
#########################Scenario 2 ########################################  
  
 **set.seed** (seed)  
  
 Dat3070<-**data.frame**(x1 = **sample**(**c**(0,1),n, replace = TRUE, prob=**c**(0.3, 0.7)),  
 x2 = **sample**(**c**(0,1),n, replace = TRUE, prob=**c**(0.3, 0.7)),  
 x3 = **sample**(**c**(0,1),n, replace = TRUE),  
 x4 = **sample**(**c**(0,1),n, replace = TRUE),  
 x5 = **sample**(**c**(0,1),n, replace = TRUE))  
 Dat3070**$**cluster <- **factor**(10000**\***(Dat3070**$**x1**+**1) **+** 1000**\***(Dat3070**$**x2**+**1) **+** 100**\***(Dat3070**$**x3**+**1) **+** 10**\***(Dat3070**$**x4**+**1) **+** 1**\***(Dat3070**$**x5**+**1))  
 Dat3070**$**int <- Dat3070**$**x1**\***Dat3070**$**x2  
 Dat3070**$**y = Dat3070**$**x1 **+** Dat3070**$**x2 **+** Dat3070**$**x3 **+** Dat3070**$**x4 **+** Dat3070**$**x5 **+** Dat3070**$**int **+** **rnorm**(n)  
   
 ###############OLS REGRESSION without interaction term  
 OLScoeff <- **summary**(**lm**(y **~** x1 **+** x2 **+** x3 **+** x4 **+** x5, data = Dat3070))**$**coefficients[,1]  
 OLScoeffSE <- **summary**(**lm**(y **~** x1 **+** x2 **+** x3 **+** x4 **+** x5, data = Dat3070))**$**coefficients[,2]  
 OLScoeff2 <- **cbind**(OLScoeff, OLScoeffSE)  
 OLScoeff2  
   
 ###############OLS REGRESSION with interaction term  
 OLSintcoeff <- **summary**(**lm**(y **~** x1 **+** x2 **+** x3 **+** x4 **+** x5 **+** x1**\***x2, data = Dat3070))**$**coefficients[,1]  
 OLSintcoeffSE <- **summary**(**lm**(y **~** x1 **+** x2 **+** x3 **+** x4 **+** x5 **+** x1**\***x2, data = Dat3070))**$**coefficients[,2]  
 OLSintcoeff2 <- **cbind**(OLSintcoeff, OLSintcoeffSE)  
 OLSintcoeff2  
   
 ###############MAIHDA, without interaction term  
 MLM1<- **brm**(formula= y **~** x1 **+** x2 **+**x3 **+** x4 **+** x5 **+** (1**|**cluster), data=Dat3070, seed=seed, cores=4, iter=50000, warmup = 5000, thin = 50)  
 *#extract main and random effects coefficients*  
 **fixef**(MLM1)  
 **ranef**(MLM1)**$**cluster  
   
 ###############MAIHDA, with interaction term  
 MLMint1<- **brm**(formula= y **~** x1 **+** x2 **+**x3 **+** x4 **+** x5 **+** x1**:**x2 **+** (1**|**cluster), data=Dat3070, seed=seed, cores=4, iter=50000, warmup = 5000, thin = 50)  
 *#extract main effects coefficients*  
 **fixef**(MLMint1)  
 **ranef**(MLMint1)**$**cluster  
  
   
  
############################## Scenario 3 ###################################  
  
  
 **set.seed** (seed)  
 Dat<-**data.frame**( x1 = **sample**(**c**(0,1),n, replace = TRUE, prob=**c**(0.8, 0.2)),  
 x2 = **sample**(**c**(0,1),n, replace = TRUE, prob=**c**(0.8, 0.2)),  
 x3 = **sample**(**c**(0,1),n, replace = TRUE),  
 x4 = **sample**(**c**(0,1),n, replace = TRUE),  
 x5 = **sample**(**c**(0,1),n, replace = TRUE))  
 Dat**$**cluster <- **factor**(10000**\***(Dat**$**x1**+**1) **+** 1000**\***(Dat**$**x2**+**1) **+** 100**\***(Dat**$**x3**+**1) **+** 10**\***(Dat**$**x4**+**1) **+** 1**\***(Dat**$**x5**+**1))  
   
 Dat**$**int <- Dat**$**x1**\***Dat**$**x2  
 Dat**$**y = Dat**$**x1 **+** Dat**$**x2 **+** Dat**$**x3 **+** Dat**$**x4 **+** Dat**$**x5 **-** (2**\***Dat**$**int) **+** **rnorm**(n)  
   
 ###############OLS REGRESSION without interaction term  
 OLScoeff <- **summary**(**lm**(y **~** x1 **+** x2 **+** x3 **+** x4 **+** x5, data = Dat))**$**coefficients[,1]  
 OLScoeffSE <- **summary**(**lm**(y **~** x1 **+** x2 **+** x3 **+** x4 **+** x5, data = Dat))**$**coefficients[,2]  
 OLScoeff2 <- **cbind**(OLScoeff, OLScoeffSE)  
 OLScoeff2  
   
 ###############OLS REGRESSION with interaction term  
 OLSintcoeff <- **summary**(**lm**(y **~** x1 **+** x2 **+** x3 **+** x4 **+** x5 **+** x1**\***x2, data = Dat))**$**coefficients[,1]  
 OLSintcoeffSE <- **summary**(**lm**(y **~** x1 **+** x2 **+** x3 **+** x4 **+** x5 **+** x1**\***x2, data = Dat))**$**coefficients[,2]  
 OLSintcoeff2 <- **cbind**(OLSintcoeff, OLSintcoeffSE)  
 OLSintcoeff2  
   
 ###############MAIHDA, without interaction term  
 MLM3<- **brm**(formula= y **~** x1 **+** x2 **+**x3 **+** x4 **+** x5 **+** (1**|**cluster), data=Dat, seed=seed, cores=4, iter=50000, warmup = 5000, thin = 50)  
 *#extract main and randoms effects coefficients*  
 **fixef**(MLM3)  
 **ranef**(MLM3)**$**cluster  
   
 ###############MAIHDA, with interaction term  
 MLMint3<- **brm**(formula= y **~** x1 **+** x2 **+**x3 **+** x4 **+** x5 **+** x1**:**x2 **+** (1**|**cluster), data=Dat, seed=seed, cores=4, iter=50000, warmup = 5000, thin = 50)  
 *#extract main and random effects coefficients*  
 **fixef**(MLMint3)  
 **ranef**(MLMint3)**$**cluster

**Results from 100 simulations**

100 simulations were conducted for each of the three scenarios. Sample sizes of 10,000 were used for each model. The multilevel models were calculated using the R brms package. Bayesian multilevel models were performed each with 1000 burn ins, 2000 total. Presented below are 0.025 and 0.975 percentiles of estimates from the 100 simulations).   
**Scenario 1:**

P(=1) = **50%;** P(=1) = **50%;** P(=1) = 50%; P(=1) = 50%; P(=1) = 50%;

Table 1. 0.025 and 0.975 percentiles of Scenario 1 from 100 simulations

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | OLS | OLS with interaction | MLM | MLM with interaction |
| Intercept | (-0.298 , -0.197) | (-0.056 , 0.052) | (-0.299 , -0.201) | (-0.056 , 0.052) |
| x1 | (1.466 , 1.538) | (0.936 , 1.056) | (1.459 , 1.534) | (0.936 , 1.056) |
| x2 | (1.454 , 1.540) | (0.948 , 1.062) | (1.458 , 1.539) | (0.948 , 1.062) |
| x3 | (0.970 , 1.037) | (0.970 , 1.040) | (0.969 , 1.039) | (0.970 , 1.041) |
| x4 | (0.966 , 1.038) | (0.968 , 1.036) | (0.969 , 1.036) | (0.968 , 1.036) |
| x5 | (0.959 , 1.042) | (0.961 , 1.038) | (0.960 , 1.039) | (0.961 , 1.039) |
| x1:x2 | - | (0.924 , 1.071) | - | (0.923 , 1.072) |

**Scenario 2:**P(=1) = **70%;** P(=1) = **70%;** P(=1) = 50%; P(=1) = 50%; P(=1) = 50%;

Table 2. 0.025 and 0.975 percentiles of Scenario 2 from 100 simulations

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | OLS | OLS with interaction | MLM | MLM with interaction |
| Intercept | (-0.552 , -0.441) | (-0.078 , 0.064) | (-0.322 , -0.208) | (-0.078 , 0.063) |
|  | (1.663 , 1.736) | (0.925 , 1.082) | (1.465 , 1.555) | (0.925 , 1.082) |
|  | (1.648 , 1.749) | (0.927 , 1.084) | (1.461 , 1.558) | (0.927 , 1.083) |
|  | (0.968 , 1.041) | (0.97 , 1.04) | (0.963 , 1.039) | (0.969 , 1.039) |
|  | (0.967 , 1.037) | (0.968 , 1.036) | (0.959 , 1.044) | (0.969 , 1.037) |
|  | (0.962 , 1.045) | (0.961 , 1.039) | (0.96 , 1.044) | (0.959 , 1.039) |
|  | - | (0.889 , 1.097) | - | (0.889 , 1.097) |

**Scenario 3:** P(=1) = **20%;** P(=1) = **20%;** P(=1) = 50%; P(=1) = 50%; P(=1) = 50%;

Table 3. 0.025 and 0.975 percentiles of Scenario 3 from 100 simulations

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | OLS | OLS with interaction | MLM | MLM with interaction |
| Intercept | (0.040 , 0.118) | (-0.039 , 0.033) | (0.432 , 0.537) | (-0.039 , 0.033) |
|  | (0.546 , 0.653) | (0.949 , 1.053) | (-0.048 , 0.077) | (0.949 , 1.054) |
|  | (0.544 , 0.657) | (0.949 , 1.056) | (-0.051 , 0.081) | (0.950 , 1.056) |
|  | (0.964 , 1.040) | (0.970 , 1.040) | (0.947 , 1.060) | (0.967 , 1.039) |
|  | (0.968 , 1.038) | (0.968 , 1.036) | (0.935 , 1.057) | (0.968 , 1.037) |
|  | (0.957 , 1.036) | (0.961 , 1.038) | (0.941 , 1.062) | (0.958 , 1.039) |
|  | - | (-2.123 , -1.889) | - | (-2.122 , -1.888) |